Pseudo-Static Deformation and Frequencies of Rotating Turbomachinery Blades

M. A. DOKAINISH
McMaster University, Hamilton, Ontario, Canada
AND
S. RAWTANI
Maulana Zzad College of Technology, Bhopal, India

Theme

IN this paper a method based on the finite element technique is developed for calculating the natural frequencies and mode shapes of rotating low aspect-ratio blades. The blade is treated as a shell. A study of the effect of different parameters such as pretwist, speed of rotation, disk radius and setting angle on the "pseudo-static deformation," caused by the centrifugal forces, and the natural frequencies of the blade, is made.

Contents

The blade is considered to be rigidly attached to the periphery of a rotating disk of radius r. A Cartesian coordinate system x, y, z is chosen for the blade with origin at the centroid of the root cross section, x axis in the radial direction and y axis parallel to the root chord. The angle between the root chord and the tangential direction of the disk is the setting angle θ. The pretwist ψ is the angle between the root chord and the tip chord of the blade. If u, v and w are the displacements along the x, y and z axes of a point on the middle surface of the blade, its instantaneous coordinates during vibration are $(x + u,\ y + v,\ z + w)$. The components of the D'Alembert force, per unit volume of the blade, are given by

$$F_x = \sum_1^4 Fx_i,\ F_y = \sum_1^4 Fy_i,\ F_z \sum_1^4 Fz_i \tag{1}$$

where

$$Fx_1 = \rho\Omega^2 (x + r),\ Fy_1 = \rho\Omega^2 (y \cos^2\theta - z \sin\theta \cos\theta) \quad Fz_1 = \rho\Omega^2 (- y \sin\theta \cos\theta + \sin^2\theta) \tag{2}$$

$$Fx_2 = \rho\Omega^2 u,\ Fy_2 = \rho\Omega^2 (v \cos^2\theta - w \sin\theta \cos\theta) \quad Fz_2 = \rho\Omega^2 (- v \sin\theta \cos\theta + w \sin^2\theta) \tag{3}$$

$$Fx_3 = - \rho\ddot{u},\quad Fy_3 = - \rho\ddot{v},\quad Fz_3 = - \rho\ddot{w} \tag{4}$$

$$Fx_4 = 2\rho\Omega(\dot{w} \sin\theta - \dot{v} \cos\theta),\quad Fy_4 = 2\rho\Omega\dot{u}\cos\theta,\quad Fz_4 = - 2\rho\Omega\dot{u} \sin\theta \tag{5}$$

in which Ω and ρ are the speed of rotation of the disk and the mass density of the blade, respectively, and the dot indicated the derivative with respect to time.

The middle surface of the blade is subdivided into a number of flat triangular elements, with nodes on the midsurface of the blade. A system of local Cartesian coordinate axes x', y', z', is chosen for each element, with nodes i, j, k numbered clockwise with the origin at node i and the axis y' along the side i-j of the element. At each node, two inplane degrees of freedom are assigned (displacements u' and v' along x' and y' axes, respectively) and three bending degrees of freedom (displacement w' along the z' axis and rotations around x' and y' axes). The displacements w', at any point (x', y') within the element, and u' and v' are assumed to be a cubic polynomial (without the terms $y'x'^2$) and linear functions in x' and y', respectively.[1]

Following the well-known finite element procedures,[2] the stiffness matrix of the complete blade is obtained. In the absence of vibrations, only centrifugal forces given by Eq. (2) are acting on the blade. These forces cause a deformation, termed pseudo-static deformation, which produces stresses in the elements, making them stiffer. This is taken into account by forming a "centrifugal stiffness matrix" and adding it to the stiffness matrix obtained earlier, thus giving the stiffness matrix $[K_t]$ of the element in the deformed position.[1,3]

For the vibration analysis, the nodal equivalent forces of the different components of the D'Alembert forces are found. For the components Fx_2, Fy_2, and Fz_2 the nodal equivalent force vector is given by $\{F\}_2 = [M_c]\{\delta\}$, where $\{\delta\}$ is a vector of nodal displacements and $[M_c]$ is the "centrifugal mass matrix." Similarly, for the components Fx_3, Fy_3 and Fz_3, $\{F\}_3 = - [M]\{\ddot{\delta}\}$, where $[M]$ is the mass matrix. The components Fx_4, Fy_4 and Fz_4 are due to Coriolis acceleration and are neglected since their effect is very small. The equation of motion for the blade is given by

$$[K_t]\{\delta\} = [M_c]\{\delta\} - [M]\{\ddot{\delta}\} \tag{6}$$

For harmonic vibrations of frequency ω, Eq. (6) reduces to

$$([K_t] - [M] - \omega^2[M])\{\delta\} = \{0\}$$

The eigenvalues of the matrix $([K_t] - [M_c])^{-1}[M]$ give the natural frequencies. The development of the matrices mentioned above is given in Ref. 3.

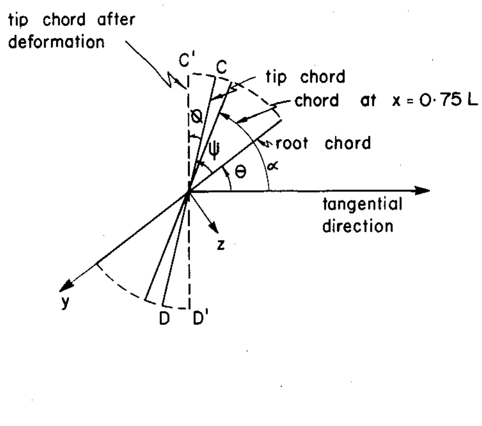

Fig. 1 Positive directions for ψ, θ, α and ϕ.

Received April 5, 1971; presented as Paper 71–374 at the AIAA/ASME 12th Structures, Structural Dynamics, and Materials Conference, Amaheim, Calif., April 19–21, 1971; synoptic received February 22, 1972; revision received May 30, 1972. Full paper is available from AIAA Library, 750 Third Avenue, New York, N.Y. 10017. Price: microfiche, $1.00; hard copy, $5.00.

Index categories: Structural Static Analysis; Structural Dynamic Analysis.

* Associate Professor of Mechanical Engineering.

† Assistant Professor in Applied Mechanics.

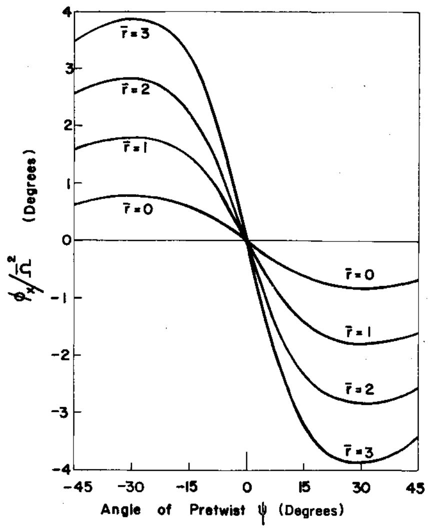

Fig. 2 Pseudo-static torsional deformation ϕ_x.

In presenting the results, the following nondimensional quantities are used

$$\bar{\Omega} = \frac{\text{Speed of rotation}}{\text{Fund. freq. of the nonrotating blade}}$$

$$\bar{r} = \frac{\text{Radius of the disk } (r)}{\text{Length of the blade } (L)}$$

$$\beta = \omega L^2 (\rho t/D)^{1/2}$$

where

$D = Et^3/12(1-\mu^2)$; t = thickness of the blade; μ = Poisson's ratio

The results are presented for an uncambered blade, with aspect ratio $(L/b) = 2$, thickness ratio $(b/t) = 16$ and $\mu = 0.3$. The examination of the pseudo-static deformations reveals that the blade undergoes a torsional deformation, in addition to a small longitudinal deformation. The total torsional deformation ϕ of the blade is given by $\phi = \phi_x + \phi_{yz}$, where ϕ_{yz} is deformation due to the components Fy_1 and Fz_1, and ϕ_x is deformation due to Fx_1. The positive direction of ψ, ϕ and θ are shown in Fig. 1. Variation of ϕ_x with ψ is shown in Fig. 2. Deformation ϕ_{yz} is observed to be given by the empirical relationship $\phi_{yz} \simeq -(\bar{\Omega}^2 \sin 2\alpha)/(a_1 + a_2\psi^2)$ where a_1 and a_2 are constants depending upon the L/b and b/t ratios of the blade, and α is the setting angle of the section of the blade at a distance of $0.75L$ from the root.

The frequency of the fundamental bending mode increases by about 90% when $\bar{r}$ increases from 0 to 2, ($\psi = 30°$, $\theta = 90°$, and $\bar{\Omega} = 1$), the corresponding increase in the second bending frequency is about 20%. The increase in the frequencies of higher bending modes is not significant. The effect of setting angle has been studied for $\psi = 30$, $\bar{r} = 2$ and $\bar{\Omega} = 1$. Only the fundamental bending mode frequency is affected, decreasing by about 10% as θ varies from 0° to 90°. The effect of pretwist is shown in Fig. 3. The effect of speed of rotation is shown in Fig. 4, the dotted curves corresponding to the case when the pseudo-static deformation is not included. In practice, the value of Ω does not exceed 1.0.

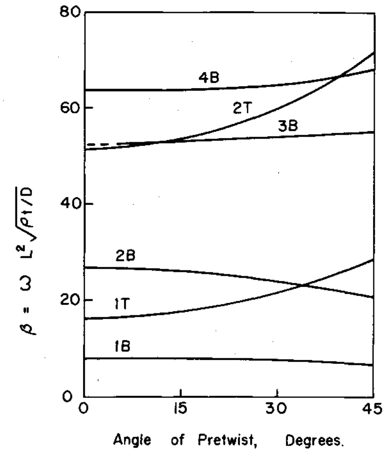

Fig. 3 Variation of natural frequencies with pretwist ($\bar{r} = 2$, $\theta = 90°$, $\bar{\Omega} = 1$).

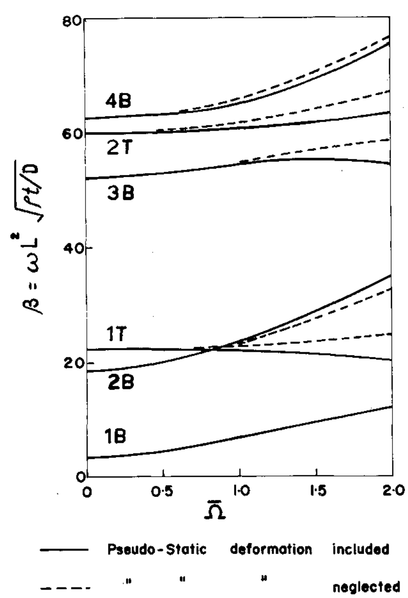

Fig. 4 Variation of natural frequencies with speed of rotation ($\bar{r} = 2$, $\psi = 30°$, $\theta = 90°$).

References

[1] Rawtani, S. and Dokainish, M. A., "Natural Frequencies of Rotating, Low Aspect Ratio, Turbomachinery Blades," AIAA paper 71-374, Anaheim, Calif. 1971.

[2] Zienkiewicz, O. C. and Cheung, Y. K., *The Finite Element Method in Structural and Continuum Mechanics*, McGraw-Hill, New York, 1967.

[3] Dokainish, M. A. and Rawtani, S., "Vibration Analysis of Rotating Cantilever Plates," *International Journal for Numerical Methods in Engineering*, Vol. 3, 1971, pp. 233–248.